Mathematics Learning Goals

Connected Mathematics develops four mathematical strands: Number and Operation, Geometry and Measurement, Data Analysis and Probability, and Algebra. The mathematical learning goals below signify what students should be able to do by the end of eighth grade in each strand. Beside each bulleted goal is a reference to the grade level (6, 7 or 8) when the specific content is covered. It is important to note that many of the goals are revisited in later units many times, either within classroom Problems or in the Connections Problems in the ACE homework assignments. For example, the bulleted goal under Number Sense of "Express rational numbers in equivalent forms" is labeled a Grade 6 goal because the unit *Bits and Pieces I* includes this goal as a "big idea". However, practice with this goal occurs throughout the curriculum.

Goals by Mathematics Strand

NUMBER AND OPERATION GOALS

Number Sense

- Use numbers in various forms to solve problems (6, 7, 8)
- Understand and use large numbers, including in exponential and scientific notation (6, 7, 8)
- Reason proportionally in a variety of contexts using geometric and numerical reasoning, including scaling and solving proportions (6, 7, 8)
- Compare numbers in a variety of ways, including differences, rates, ratios, and percents and choose when each comparison is appropriate (6, 7, 8)
- Order positive and/or negative rational numbers (6, 7, 8)
- Express rational numbers in equivalent forms (6)
- Make estimates and use benchmarks (6, 7, 8)

Operations and Algorithms

- Develop understanding and skill with all four arithmetic operations on fractions and decimals (6)
- Develop understanding and skill in solving a variety of percent problems (6)
- Use the order of operations to write, evaluate, and simplify numerical expressions (7, 8)
- Develop fluency with paper and pencil computation, calculator use, mental calculation, and estimation; and choose among these when solving problems (6, 7)

Properties

- Understand the multiplicative structure of numbers, including the concepts of prime and composite numbers, evens, odds, and prime factorizations (6)
- Use the commutative and distributive properties to write equivalent numerical expressions (7, 8)

DATA AND PROBABILITY GOALS

Formulating Questions

- Formulate questions that can be answered through data collection and analysis (6, 7, 8)
- Design data collection strategies to gather data to answer these questions (6, 7, 8)
- Design experiments and simulations to test hypotheses about probability situations (8)

Data Collection

- Carry out data collection strategies to answer questions (6, 7, 8)
- Distinguish between samples and populations (8)
- Characterize samples as representative or non-representative, as random (8)
- Use these characterizations to evaluate the quality of the collected data (8)

Data Analysis

- Organize, analyze, and interpret data to make predictions, construct arguments, and make decisions (6, 7)
- Use measures of center and spread to describe and to compare data sets (6, 7)
- Be able to read, create, and choose data representations, including bar graphs, line plots, coordinate graphs, box and whisker plots, histograms, and stem and leaf plots (6, 7)
- Informally evaluate the significance of differences between sets of data (7, 8)
- Use information from samples to draw conclusions about populations (8)

Probability

- Distinguish between theoretical and experimental probabilities and understand the relationship between them (6)
- Use probability concepts to make decisions (6)
- Find and interpret expected value (7)
- Compute and compare the chances of various outcomes, including two-stage outcomes (7)

GEOMETRY AND MEASUREMENT GOALS

Shapes and Their Properties

- Generate important examples of angles, lines, and two- and three-dimensional shapes (6)
- Categorize, define, and relate figures in a variety of representations (6, 7)
- Understand principles governing the construction of shapes with reasons why certain shapes serve special purposes (e.g. triangles for trusses) (6)
- Build and visualize three-dimensional figures from various two-dimensional representations and vice versa (7)
- Recognize and use shapes and their properties to make mathematical arguments and to solve problems (6, 7, 8)
- Use the Pythagorean Theorem and properties of special triangles (e.g. isosceles right triangles) to solve problems (8)
- Use a coordinate grid to describe and investigate relationships among shapes (7, 8)
- Recognize and use standard, essential geometric vocabulary (6, 7, 8)

Transformations—Symmetry, Similarity, and Congruence

- Recognize line, rotational, and translational symmetries and use them to solve problems (6, 8)
- Use scale factor and ratios to create similar figures or determine whether two or more shapes are similar or congruent (7)
- Predict ways that similarity and congruence transformations affect lengths, angle measures, perimeters, areas, volume, and orientation (7, 8)
- Investigate the effects of combining one or more transformations of a shape (8)
- Identify and use congruent triangles and/or quadrilaterals to solve problems about shapes and measurement (6, 8)
- Use properties of similar figures to solve problems about shapes and measurement (7)
- Use a coordinate grid to explore and verify similarity and congruence relationships (7, 8)

Measurement

- Understand what it means to measure an attribute of a figure or a phenomenon (6)
- Estimate and measure angles, line segments, areas, and volumes using tools and formulas (6, 7)
- Relate angle measure and side lengths to the shape of a polygon (6)
- Find area and perimeter of rectangles, parallelograms, triangles, circles, and irregular figures (7)
- Find surface area and volume of rectangular solids, cylinders, prisms, cones, and pyramids and the volume of spheres (7)
- Relate units within and between the customary and metric systems (6, 7)
- Use ratios and proportions to derive indirect measurements (7)
- Use measurement concepts to solve problems (6, 7, 8)

Geometric Connections

- Use geometric concepts to build understanding of concepts in other areas of mathematics (6, 7, 8)
- Connect geometric concepts to concepts in other areas of mathematics (6, 7, 8)

ALGEBRA GOALS

Patterns of Change—Functions

- Identify and use variables to describe relationships between quantitative variables in order to solve problems or make decisions (7, 8)
- Recognize and distinguish among patterns of change associated with linear, inverse, exponential and quadratic functions (7, 8)

Representation

- Construct tables, graphs, symbolic expressions and verbal descriptions and use them to describe and predict patterns of change in variables (7, 8)
- Move easily among tables, graphs, symbolic expressions, and verbal descriptions (7, 8)
- Describe the advantages and disadvantages of each representation and use these descriptions to make choices when solving problems (7, 8)
- Use linear, inverse, exponential, and quadratic equations and inequalities as mathematical models of situations involving variables (7, 8)

Symbolic Reasoning

- Connect equations to problem situations (7, 8)
- Connect solving equations in one variable to finding specific values of functions (8)
- Solve linear equations and inequalities and simple quadratic equations using symbolic methods (7, 8)
- Find equivalent forms of many kinds of equations, including factoring simple quadratic equations (7, 8)
- Use the distributive and commutative properties to write equivalent expressions and equations (8)
- Solve systems of linear equations (8)
- Solve systems of linear inequalities by graphing (8)

Content Goals in Each Unit

Connected Mathematics 2 provides eight student units for each grade level. Each unit is organized around an important mathematical idea or cluster of related ideas as described in the table on page 15. Each unit covers material in a particular strand of mathematics. This classification by strand is meant to highlight the strand that is the primary focus of the unit. However, there are problems in every unit that connect to the other three strands. For example, the unit *Shapes of Algebra* is classified under Algebra. Even though this unit's focus is primarily on algebraic ideas, there are many connections to geometry, as the unit's name implies.

UNITS ORGANIZED BY STRAND

Number

Prime Time (Grade 6)
Bits and Pieces I (Grade 6)
Bits and Pieces II (Grade 6)
Bits and Pieces III (Grade 6)
Comparing and Scaling (Grade 7)
Accentuate the Negative (Grade 7)

Geometry

Shapes and Designs (Grade 6)
Covering and Surrounding (Grade 6)
Stretching and Shrinking (Grade 7)
Filling and Wrapping (Grade 7)
Looking of Pythagoras (Grade 8)
Kaleidoscopes, Hubcaps, and Mirrors (Grade 8)

Algebra

Variables and Patterns (Grade 7)
Moving Straight Ahead (Grade 7)
Thinking With Mathematical Models (Grade 8)
Growing, Growing, Growing (Grade 8)
Frogs, Fleas, and Painted Cubes (Grade 8)
Say It With Symbols (Grade 8)
Shapes of Algebra (Grade 8)

Data Analysis and Probability

How Likely Is It? (Grade 6)
Data About Us (Grade 6)
What Do You Expect? (Grade 7)
Data Distributions (Grade 7)
Samples and Populations (Grade 8)

In order to have a clearer idea of the particular goals for each unit, the goals are listed below by unit name. The units are sequenced in the order they are intended to be taught.

The ninth unit at each grade level, available from the first edition of CMP, can be used as a stand-alone unit for various purposes. For example, the unit *Ruins of Montarek* has been taught at Grade 5, as well as in art classes and social studies classes. Depending on your state or local standards, parts or all of these three additional units can be supplemented into the curriculum. The goals of each supplemental unit are listed below, after the CMP2 units.

Some questions to ask yourself as you examine the list of unit goals that follow:

- *How does a particular strand play out? For example, how are the number units sequenced? What units in each grade are in the number strand? How do the number systems with which students work grow as the curriculum progresses (ie, whole numbers, fractions, decimals, irrational numbers)*
- *When following a key goal for a unit: Does a later unit further develop this same goal and if so how?*
- *What goals have my students already met from prior units? What prior knowledge do they have that I can draw on?*
- *How does a concept grow? For example, which units are setting the groundwork for linear functions? What units cover this topic and how does this idea grow in complexity?*
- *Why are the units from different strands interspersed? What connections are made between the strands, between the units within a grade level, between the units in different grade levels?*

GRADE SIX GOALS

Prime Time (Number)

- Understand relationships among factors, multiples, divisors, and products
- Recognize and use properties of prime and composite numbers, even and odd numbers, and square numbers
- Use rectangles to represent the factor pairs of numbers
- Develop strategies for finding factors and multiples, least common multiples, and greatest common factors
- Recognize and use the fact that every whole number greater than 1 can be written in exactly one way as a product of prime numbers
- Use factors and multiples to solve problems and to explain some numerical facts of everyday life
- Develop a variety of strategies for solving problems—building models, making lists and tables, drawing diagrams, and solving simpler problems

Bits and Pieces I (Number)

- Build an understanding of fractions, decimals, and percents and the relationships between and among these concepts and their representations
- Develop ways to model situations involving fractions, decimals, and percents
- Understand and use equivalent fractions to reason about situations
- Compare and order fractions
- Move flexibly among fraction, decimal, and percent representations
- Use benchmarks such as 0, 1/2 and 1 to help estimate the size of a number or sum
- Develop and use benchmarks that relate different forms of representations of rational numbers (for example, 50% can be represented as 0.5)
- Use physical models and drawings to help reason about a situation
- Look for patterns and describe how to continue the pattern
- Use context to help reason about a situation
- Use estimation to understand a situation

Shapes and Designs (Geometry)

- Understand some important properties of polygons and recognize polygonal shapes both in and out of the classroom
- Investigate the symmetries of a shape—rotational or reflectional
- Estimate the size of any angle using reference to a right angle and other benchmark angles
- Use an angle ruler for making more accurate angle measurements
- Explore parallel lines and angles created by lines intersecting parallel lines

- Find patterns that help determine angle sums of polygons
- Determine which polygons fit together to cover a flat surface and why
- Explain the property of triangles that makes them useful as a stable structure for building
- Find that the sum of any two side lengths of a triangle is greater than the third side length
- Find that the sum of any three side lengths of a quadrilateral is greater than the fourth side length
- Reason about and solve problems involving shapes

Bits and Pieces II (Number)

- Use benchmarks and other strategies to estimate the reasonableness of results of operations with fractions
- Develop ways to model sums, differences, products, and quotients with areas, strips, and number lines
- Use estimates and exact solutions to make decisions
- Look for and generalize patterns in numbers
- Use knowledge of fractions and equivalence of fractions to develop algorithms for adding, subtracting, multiplying and dividing fractions
- Recognize when addition, subtraction, multiplication, or division is the appropriate operation to solve a problem
- Write fact families to show the inverse relationship between addition and subtraction, and between multiplication and division
- Solve problems using arithmetic operations on fractions

Covering and Surrounding (Geometry)

- Understand area and relate area to covering a figure
- Understand perimeter and relate perimeter to surrounding a figure
- Develop strategies for finding areas and perimeters of rectangular shapes and non-rectangular shapes
- Discover relationships between perimeter and area. including that each can vary while the other stays fixed
- Understand how the areas of simple geometric figures relate to each other (e.g. the area of a parallelogram is twice the area of a triangle with the same base and height)
- Develop formulas and procedures—stated in words and/or symbols—for finding areas and perimeters of rectangles, parallelograms, triangles, and circles
- Develop techniques for estimating the area and perimeter of an irregular figure
- Recognize situations in which measuring perimeter or area will help answer practical questions

Bits and Pieces III (Number)

- Build on knowledge of operations on fractions and whole numbers
- Develop and use benchmarks and other strategies to estimate the answers to computations with decimals
- Develop meaning of and algorithms for operations with decimals
- Use the relationship between decimals and fractions to develop and understand why decimal algorithms work
- Use the place value interpretation of decimals to make sense of short-cut algorithms for operations
- Generalize number patterns to help make sense of decimal operations
- Choose between addition, subtraction, multiplication or division as an appropriate operation to use to solve a problem
- Understand that decimals are often associated with measurements in real world situations
- Solve problems using operations on decimals
- Use understanding of operations and the meaning of percents to solve percent problems of the form a% of b equals c for any one of the variables a, b, or c
- Create and interpret circle graphs

How Likely Is It? (Probability)

- Understand that probabilities are useful for predicting what will happen over the long run
- Understand the concepts of equally likely and not-equally likely
- Understand that fairness implies equally likely outcomes

- Understand that there are two ways to build probability models: by gathering data from experiments (experimental probability) and by analyzing the possible equally likely outcomes (theoretical probability)
- Understand that experimental probabilities are better estimates of theoretical probabilities when they are based on larger numbers of trials
- Develop strategies for finding both experimental and theoretical probabilities
- Critically interpret statements of probability to make decisions or answer questions

Data About Us (Data Analysis)

- Understand and use the process of data investigation by posing questions, collecting data, analyzing data distributions, and making interpretations to answer questions
- Represent data distributions using line plots, bar graphs, stem-and-leaf plots, and coordinate graphs
- Compute the mean, median, or mode and the range of the data
- Distinguish between categorical data and numerical data and identify which graphs and statistics may be used to represent each kind of data
- Make informed decisions about which graph or graphs and which of the measures of center (mean, median, or mode) and range may be used to describe a data distribution
- Develop strategies for comparing data distributions

Ruins of Montarek (Geometry)

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- Read and make two-dimensional representations of three-dimensional cube buildings
- Observe that the back view of a cube building is the mirror image of the front view and that the left view is the mirror image of the right view
- Explain how drawings of the base outline, front view, and right view describe a building
- Construct cube buildings that fit two-dimensional building plans
- Develop a way to describe all buildings that can be made from a set of plans
- Understand that a set of plans can have more than one minimal building but only one maximal building
- Explain how a cube can be represented on isometric dot paper, how the angles on the cube are represented with angles on the dot paper, and how the representations fit what the eye sees when viewing the corner of a cube building
- Make isometric drawings of cube buildings
- Visualize transformations of cube buildings and make isometric drawings of the transformed buildings
- Reason about spatial relationships
- Use models and representations to solve problems

GRADE SEVEN GOALS

Variables and Patterns (Algebra)

- Recognize problem situations in which two or more quantitative variables are related to each other
- Identify quantitative variables in situations
- Describe patterns of change between two variables that are shown in words, tables and graphs of data
- Construct tables and graphs to display relations among variables
- Observe relationships between two variables as shown in a table, graph, or equation and describe how the relationship can be seen in each of the other forms of representation
- Use algebraic symbols to write equations relating variables
- Use tables, graphs, and equations to solve problems
- Use graphing calculators to construct tables and graphs of relations between variables and to answer questions about these relations

Stretching and Shrinking (Geometry)

- Identify similar figures by comparing corresponding parts
- Use scale factors and ratios to describe relationships among the side lengths of similar figures
- Construct similar polygons
- Draw shapes on coordinate grids and then use coordinate rules to stretch and shrink those shapes
- Predict the ways that stretching or shrinking a figure affect lengths, angle measures, perimeters, and areas

- Use the properties of similarity to calculate distances and heights that can't be directly measured

Comparing and Scaling (Number)

- Analyze comparison statements made about quantitative data
- Use ratios, fractions, differences, and percents to form comparison statements in a given situation, such as

  "What is the ratio of boys to girls in our class?"

  "What fraction of the class is going to the spring picnic?"

  "What percent of the girls play basketball?"

  "Which model of car has the best fuel economy?"

  "Which long-distance telephone company is more popular?"
- Judge whether comparison statements make sense and are useful
- See how forms of comparison statements are related, for example, a percent and a fraction comparison
- Make judgments about which statements are most informative or best reflect a particular point of view
- Decide when the most informative comparison is to find the difference between two quantities and when it is to form ratios between pairs of quantities
- Scale a ratio, rate, or fraction up or down to make a larger or smaller object or population with the same relative characteristics as the original
- Represent related data in tables
- Look for patterns in tables that will allow predictions to be made beyond the tables
- Write an equation to represent the pattern in a table of related variables
- Apply proportional reasoning to solve for the unknown part when one part of two equal ratios is unknown
- Set up and solve proportions that arise in applications
- Recognize that constant growth in a table is related to proportional situations
- Connect unit rates with the equation describing a situation

Accentuate the Negative (Number)

- Use appropriate notation to indicate positive and negative numbers
- Locate rational numbers (positive and negative fractions and decimals and zero) on a number line
- Compare and order rational numbers
- Understand the relationship between a positive or negative number and its opposite (additive inverse)
- Develop algorithms for adding, subtracting, multiplying, and dividing positive and negative numbers and write mathematics sentences to show relationships
- Write and use related fact families for addition/subtraction and multiplication/division to solve simple equations with missing facts
- Use parentheses and order of operations to make computational sequences clear
- Understand and use the Commutative Property for addition and multiplication of negative and positive numbers
- Apply the Distributive Property with positive and negative numbers to simplify expressions and solve problems
- Use positive and negative numbers to graph in four quadrants, model and answer questions about applied settings

Moving Straight Ahead (Algebra)

- Recognize problem situations in which two or more variables have a linear relationship to each other
- Construct tables, graphs, and symbolic equations that express linear relationships
- Translate information about linear relations given in a table, a graph, or an equation to one of the other forms
- Understand the connections between linear equations and patterns in the tables and graphs of those relations—rate of change, slope, and y-intercept
- Solve linear equations
- Solve problems and make decisions about linear relationships using information given in tables, graphs, and symbolic expressions
- Use tables, graphs, and equations of linear relations to answer interesting questions

Filling and Wrapping (Geometry)

- Understand volume as a measure of filling an object and surface area as a measure of wrapping or covering an object
- Use flat patterns to visualize and calculate surface areas of prisms and cylinders
- Develop formulas for the volumes of prisms, cylinders, cones, pyramids, and spheres either directly or by comparison with known volumes
- Understand that three-dimensional figures may have the same volume but quite different shapes and surface areas or that they may have the same surface area but different shapes and volumes
- Use surface area and volume to solve a variety of real-world problems
- Understand how changes in one or more dimensions of a rectangular prism or cylinder affects the prism's volume
- Extend students' understanding of similarity and scale factors to three-dimensional figures
- Understand the effect on surface area and volume of applying a scale factor to a rectangular prism

Data Distributions (Data Analysis)

- Apply the process of statistical investigation to pose questions, identify ways data are collected, determine strategies for analyzing data and interpreting the analysis in order to answer the questions posed
- Compare the distributions of data using their related centers, variability, and shapes
- Use the shape of a distribution to estimate the mean and median
- Recognize that variability occurs whenever data are collected and use properties of distributions to describe the variability in a given data set
- Identify sources of variability, including natural variability and variability that results from errors in measurement
- Decide if a difference among data values and/or summary measures matters
- Understand and decide when to use the mean and median to describe a distribution
- Make effective use of a variety of representations to display distributions, including tables, value bar graphs, dot or line plots, and bar graphs
- Understand and use counts or percents to report frequencies of occurrence of data
- Develop and use strategies for comparing equal-size and unequal-size data sets to solve problems

What Do You Expect? (Probability)

- Interpret experimental and theoretical probabilities and the relationship between them
- Distinguish between equally likely and non-equally likely outcomes
- Review strategies for identifying possible outcomes and analyzing probabilities, such as using lists or counting trees
- Understand that fairness implies equally likely outcomes
- Analyze situations that involve two-stages (or two actions)
- Use area models to analyze situations that involve two stages
- Determine the expected value of a probability situation
- Analyze binomial situations
- Use probability and expected value to make decisions

Numbers Around Us (Number)

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- Choose sensible units for measuring
- Understand that a measurement has two components, a unit of measure and a count
- Build a repertoire of benchmarks to relate the measures of unfamiliar objects or events to the measures of objects or events that are personally meaningful
- Review the concept of place value as it relates to reading, writing, and using large numbers
- Read, write, and interpret the large numbers that occur in real-life measurements using standard, scientific, and calculator notation
- Review and extend the use of exponents
- Use estimates and rounded values for describing and comparing objects and events
- Develop strategies for operating with large numbers
- Choose sensible ways of comparing counts and measurements, including using differences, rates, and ratios
- Draw sensible conclusions from given information.

GRADE EIGHT GOALS

Thinking With Mathematical Models (Algebra)

- Recognize linear and non-linear patterns in contexts, tables and graphs and describe those patterns using words and symbolic expressions
- Write equations to express linear patterns appearing in tables, graphs, and verbal contexts
- Write linear equations when specific information such as two points or a point and a slope, is given for a line
- Approximate linear data patterns with graph and equation models
- Solve linear equations
- Interpret inequalities
- Write equations describing inverse variation
- Use linear and inverse variation equations to solve problems and to make predictions and decisions

Looking For Pythagoras (Algebra)

- Relate the area of a square to the length of a side of the square
- Estimate square roots
- Develop strategies for finding the distance between two points on a coordinate grid
- Understand and apply the Pythagorean Theorem
- Use the Pythagorean Theorem to solve a variety of problems

Growing, Growing, Growing (Algebra)

- Recognize situations where one variable is an exponential function of another variable
- Recognize the connections between exponential equations and growth patterns in tables and graphs of those relations
- Construct equations to express exponential patterns that appear in data tables, graphs, and problem conditions
- Understand and apply the rules for operating on numerical expressions with exponents
- Solve problems about exponential growth and decay in a variety of situations such as science or business
- Compare exponential and linear relationships

Frogs, Fleas, and Painted Cubes (Algebra)

- Recognize the patterns of change for quadratic relationships in a table, graph, equation, and problem situation
- Construct equations to express quadratic relationships that appear in tables, graphs and problem situations
- Recognize the connections between quadratic equations and patterns in tables and graphs of those relationships
- Use tables, graphs, and equations of quadratic relationships to locate maximum and minimum values of a dependent variable and the x- and y-intercepts and other important features of parabolas
- Recognize equivalent symbolic expressions for the dependent variable in quadratic relationships
- Use the distributive property to write equivalent quadratic expressions in factored form or expanded form
- Use tables, graphs, and equations of quadratic relations to solve problems in a variety of situations from geometry, science, and business
- Compare properties of quadratic, linear, and exponential relationships

Kaleidoscopes, Hubcaps, and Mirrors (Geometry)

- Understand important properties of symmetry
- Recognize and describe symmetries of figures
- Use tools to examine symmetries and transformations
- Make figures with specified symmetries
- Identify basic design elements that can be used to replicate a given design
- Perform symmetry transformations of figures, including reflections, translations, and rotations
- Examine and describe the symmetries of a design made from a figure and its image(s) under a symmetry transformation
- Give precise mathematical directions for performing reflections, rotations, and translations
- Draw conclusions about a figure, such as measures of sides and angles, lengths of diagonals, or intersection points of diagonals, based on symmetries of the figure

- Understand that figures with the same shape and size are congruent
- Use symmetry transformations to explore whether two figures are congruent
- Give examples of minimum sets of measures of angles and sides that will guarantee that two triangles are congruent
- Use congruence of triangles to explore congruence of two quadrilaterals
- Use symmetry and congruence to deduce properties of figures
- Write coordinate rules for specifying the image of a general point (x, y) under particular transformations
- Use transformational geometry to describe motions, patterns, designs, and properties of shapes in the real world

Say It With Symbols (Algebra)

- Model situations with symbolic statements
- Write equivalent expressions
- Determine if different symbolic expressions are mathematically equivalent
- Interpret the information equivalent expressions represent in a given context
- Determine which equivalent expression to use to answer particular questions;
- Solve linear equations involving parentheses
- Solve quadratic equations by factoring
- Use equations to make predictions and decisions
- Analyze equations to determine the patterns of change in the tables and graphs that the equation represents
- Understand how and when symbols should be used to display relationships, generalizations, and proofs

The Shapes of Algebra (Algebra)

- Write and use equations of circles
- Determine lines are parallel or perpendicular by looking at patterns in their graphs, coordinates, and equations
- Find coordinates of points that divide line segments in various ratios
- Write inequalities that satisfy given situations
- Find solutions to inequalities represented by a graph or an equation
- Solve systems of linear equations by graphing, combining equations, and by substitution
- Write linear inequalities in two variables to match constraints in problem conditions
- Graph linear inequalities and systems of inequalities and use the results to solve problems

Samples and Populations (Data Analysis)

- Revisit and use the process of statistical investigation to explore problems
- Distinguish between samples and populations and use information drawn from samples to draw conclusions about populations
- Explore the influence of sample size and of random or nonrandom sample selection
- Apply concepts from probability to select random samples from populations
- Compare sample distributions using measures of center (mean or median), measures of dispersion (range or percentiles), and data displays that group data (histograms and box-and-whisker plots)
- Explore relationships between paired values of numerical attributes

Clever Counting (Number)

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- Recognize situations in which counting techniques apply
- Construct organized lists of outcomes for complex processes and uncover patterns that help in counting the outcomes of those processes
- Use diagrams, tables, and symbolic expressions to organize examples in listing and counting tasks
- Analyze the usefulness of counting trees and use counting trees
- Use mental arithmetic to make estimates in multiplication and division calculations
- Invent strategies for solving problems that involve counting
- Analyze counting problems involving choices in various contexts
- Differentiate among situations in which order does and does not matter and in which repeats are and are not allowed

- Analyze the number of paths through a network
- Compare the structures of networks with problems involving combinations
- Create networks that satisfy given constraints
- Apply thinking and reasoning skills to an open-ended situation in which assumptions must be made and create a persuasive argument to support a conjecture

Mathematics Process Goals

In setting mathematical goals for a school curriculum, the choice of content topics must be accompanied by an analysis of the kinds of thinking students will be able to demonstrate upon completion of the curriculum. The text below describes the eleven key mathematical processes developed in all the main content strands of *Connected Mathematics*.

Counting

Determining the number of elements in finite data sets, trees, graphs, or combinations by application of mental computation, estimation, counting principles, calculators and computers, and formal algorithms.

Visualizing

Recognizing and describing shape, size, and position of one-, two-, and three-dimensional objects and their images under transformations; interpreting graphical representations of data, functions, relations, and symbolic expressions.

Comparing

Describing relationships among quantities and shapes using concepts such as equality and inequality, order of magnitude, proportion, congruence, similarity, parallelism, perpendicularity, symmetry, and rates of growth or change.

Estimating

Determining reasonableness of answers. Using "benchmarks" to estimate measures. Using various strategies to approximate a calculation and to compare estimates.

Measuring

Assigning numbers as measures of geometric objects and probabilities of events. Choosing appropriate measures in a decision-making problem. Choosing appropriate units or scales and making approximate measurements or applying formal rules to find measures.

Modeling

Constructing, making inferences from, and interpreting concrete, symbolic, graphic, verbal, and algorithmic models of quantitative, visual, statistical, probabilistic, and algebraic relationships in problem situations. Translating information from one model to another.

Reasoning

Bringing to any problem situation the disposition and ability to observe, experiment, analyze, abstract, induce, deduce, extend, generalize, relate, and manipulate in order to find solutions or prove conjectures involving interesting and important patterns.

Connecting

Identifying ways in which problems, situations, and mathematical ideas are interrelated and applying knowledge gained in solving one problem to other problems.

Representing

Moving flexibly among graphic, numeric, symbolic, and verbal representations and recognizing the importance of having various representations of information in a situation.

Using Tools

Selecting and intelligently using calculators, computers, drawing tools, and physical models to represent, simulate, and manipulate patterns and relationships in problem settings.

Becoming Mathematicians

Having the disposition and imagination to inquire, investigate, tinker, dream, conjecture, invent, and communicate with others about mathematical ideas.